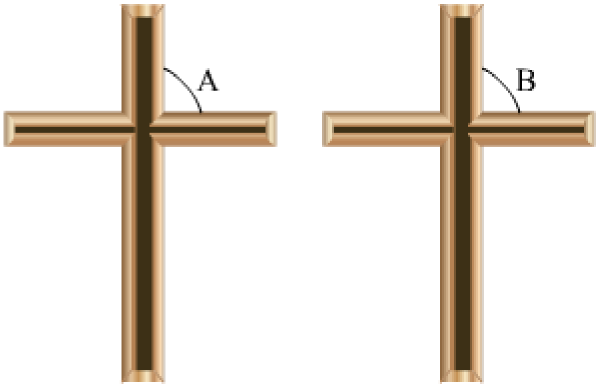
Lines and Angles

Corresponding Angles Axiom

Corresponding Angles in Real Life

Consider the two crosses shown in the given figure.



Cross I Cross II

In cross I, the right arm makes ∠A with the head of the cross; in cross II, the right arm makes ∠B with the head of the cross. Thus, both ∠A and ∠B are at the same position in the two crosses. Such angles are called **corresponding angles**.

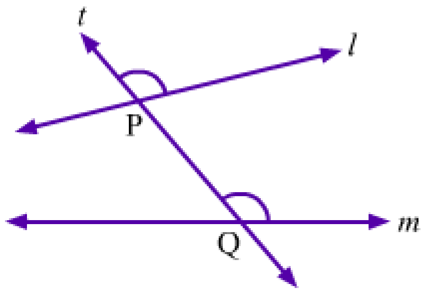
Now, suppose the right arm of cross I is joined to the left arm of cross II. What we obtain is a transversal cutting across a pair of lines to form different pairs of corresponding angles (like ∠A and ∠B). So, we can say that the relation between the angles in the so formed corresponding pairs of angles is decided by the relation between the lines cut by the transversal. This is defined by the **corresponding angles axiom**.

In this lesson, we will discuss the above axiom and its converse. We will also crack some problems based on the same.

Concept Builder

A transversal is a line that intersects two or more lines in the same plane at different points.

Look, for example, at the following figure.



In the figure, line *t* intersects lines *l* and *m* at two different points P and Q respectively; so, *t* is a transversal. The marked angles are corresponding angles.

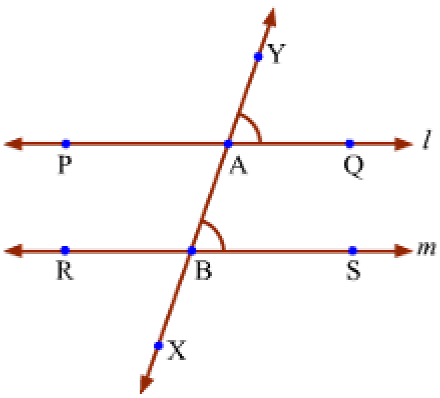
Proof of Corresponding Angle Axiom

When a transversal crosses two parallel lines, each pair of corresponding angles is equal (or congruent).

Given: Transversal XY of coplanar lines *l* and *m* such that *l* || *m*

To prove: ∠YAQ = ∠ABS

Proof:



l || *m* (Given)

∴ ∠PAB = ∠ABS ...(1) (Alternate angles)

And, ∠PAB = ∠YAQ ...(2) (Vertically opposite angles)

From (1) and (2), we obtain

∠YAQ = ∠ABS

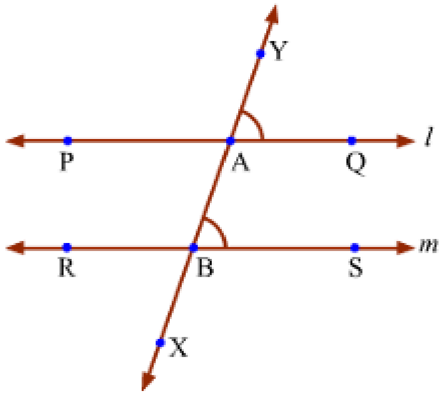
Proof of Converse of Corresponding Angle Axiom

If a pair of corresponding angles is equal (or congruent) then the lines are parallel to each other.

Given: Transversal XY of coplanar lines *l* and *m* such that ∠YAQ = ∠ABS

To prove: *l* || *m*

Proof:



∠YAQ = ∠ABS ...(1) (Given)

And, ∠YAQ = ∠PAB ...(2) (Vertically opposite angles)

From (1) and (2), we obtain

∴ ∠PAB = ∠ABS

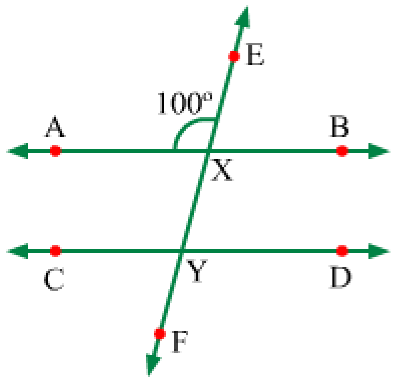
∴ *l* || *m* (Using alternate angle axiom)

Note: If one pair of corresponding angles is equal (or congruent) then all the pairs of corresponding angles are equal.

Solved Examples

Easy

Example 1: In the given figure, if AB and CD are parallel lines, then find the measure of ∠CYX.



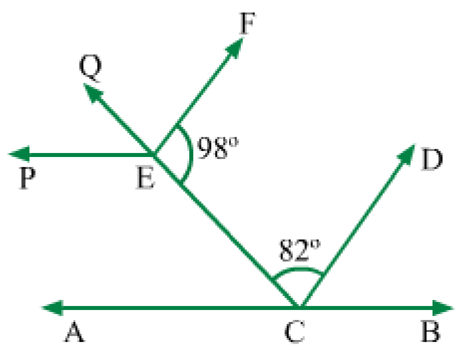
Solution:

In the figure, ∠AXE and ∠CYX are corresponding angles made by the transversal EF on lines AB and CD respectively.

Using the corresponding angles axiom, we obtain:

∠CYX = ∠AXE = 100°

Example 2: **In the given figure, prove that EF||CD.**



Solution:

In the give figure, ∠QEF and ∠FEC form a linear pair.

⇒ ∠QEF + ∠FEC = 180°

⇒ ∠QEF + 98° = 180°

⇒ ∴ ∠QEF = 82°

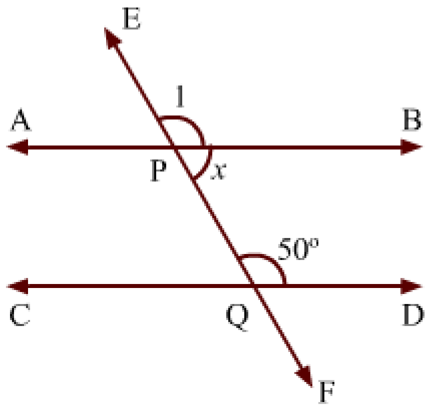
It can be seen that ∠QEF and ∠ECD are corresponding angles made by the transversal QC on lines EF and CD respectively.

Now, ∠QEF = ∠ECD = 82°

∴ EF||CD (By the converse of the corresponding angles axiom)

Medium

Example 1: **In the given figure, line segments AB and CD are parallel. Find the value of *x*.**



Solution:

It is given that AB is parallel to CD and EF is the transversal.

∴ ∠1 = ∠DQP = 50° (Corresponding angles)

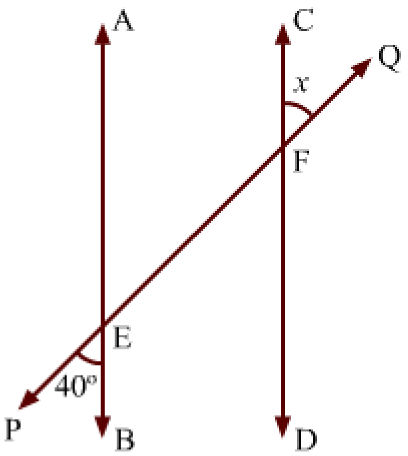
Now, ∠1 + x = 180° (Angles forming a linear pair)

⇒ 50° + x = 180°

⇒ x = 180° − 50°

⇒ ∴ x = 130°

Example 2: **In the given figure, line segments AB and CD are parallel. What is the value of *x*?**



Solution:

It is given that AB is parallel to CD and PQ is the transversal.

∠PEB = 40° (Given)

∠AEF = ∠PEB = 40° (Vertically opposite angles)

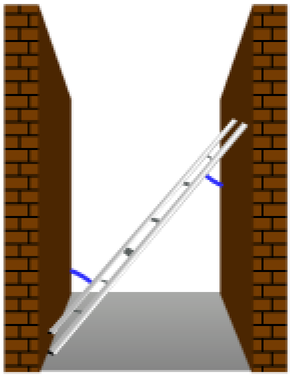
x = ∠AEF (By the corresponding angles axiom)

⇒ ∴ x = 40°

Alternate Angle Axiom

Introduction to Alternate Angles

We can find examples of alternate angles in daily life. These angles play an important role in various situations. Look, for example, at the given figure.

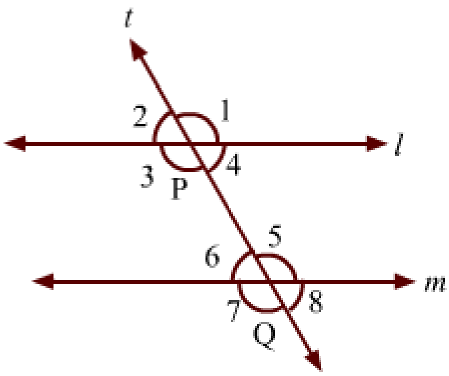


In the figure, the ladder resting between the two walls resembles a transversal joining two lines. The angles marked in the figure are **alternate angles**. These angles exhibit a special property that is defined by the **alternate angles axiom**.

In this lesson, we will study about the above axiom and its converse. We will also solve some examples based on the same.

Alternate Angles Axiom

Consider the given figure in which line *t* is a transversal intersecting two parallel lines *l* and *m* at points P and Q respectively.



In the figure, there are two pairs of alternate angles lying outside the parallel lines, i.e., ∠1 and ∠7, and ∠2 and ∠8. These angles are called **alternate exterior angles**. Also, there are two pairs of alternate angles lying between the parallel lines, i.e., ∠3 and ∠5, and ∠4 and ∠6. These angles are called **alternate interior angles**. The alternate angles made by a transversal on parallel lines have a special property which is stated as follows:

If a transversal intersects two parallel lines, then the angles in each pair of alternate angles are equal.

This property is known as the alternate angles axiom.

So, by using the alternate angles axiom, we can say the following for the given figure.

∠1 = ∠7, ∠2 = ∠8, ∠3 = ∠5 and ∠4 = ∠6

The converse of this axiom is also true. It states that:

If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.

The alternate angles axiom and its converse are helpful in solving many problems in geometry as well as in real life.

Did You Know?

Playfair's axiom

If *m* is a line and P is any point which does not lie on this line, then there is only one line parallel to line *m* through point P.

For example:

In the given figure, line *l* is the only line parallel to line *m* through point P.

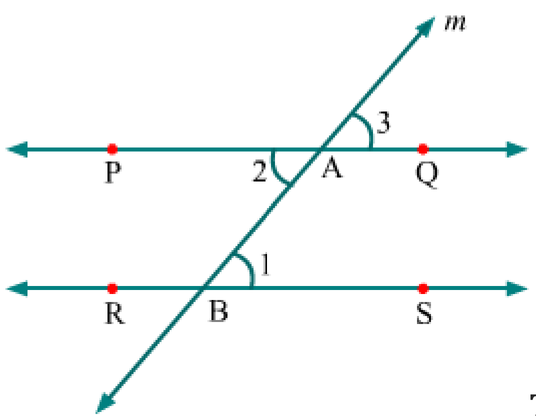
Proof of the Converse of the Alternate Angles Axiom

Statement of the converse of the alternate angles axiom

If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.

Let us prove this statement.

Given: The transversal *m* intersects lines PQ and RS at points A and B respectively, such that the alternate angles 1 and 2 are equal.



To prove: PQ||RS

Proof: From the figure, we have

∠2 = ∠3 (Vertically opposite angles)

∠1 = ∠2 (Given)

⇒ ∠1 = ∠3

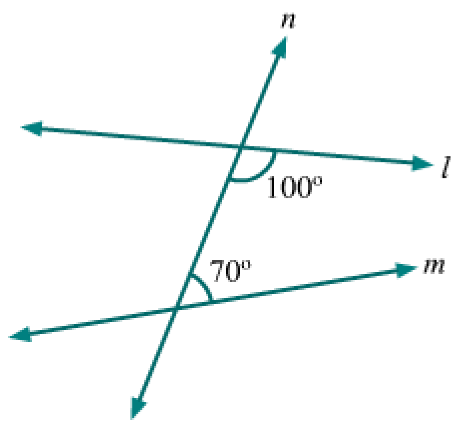
∠1 and ∠3 are corresponding angles; so, by using the converse of the corresponding angles axiom, it can be said that PQ and RS are parallel or PQ||RS.

Note: If one pair of alternate angles is equal (or congruent) then the other pair of alternate angles is also equal (or congruent).

Whiz Kid

The great mathematician Euclid gave a few important results about geometry which are known as Euclid's postulates. The fifth postulate (or the parallel postulate) is about transversals. This postulate states that *if the sum of the interior angles on the same side of a transversal is less than two right angles, then the lines cut by the transversal must intersect when extended indefinitely*.

For example:

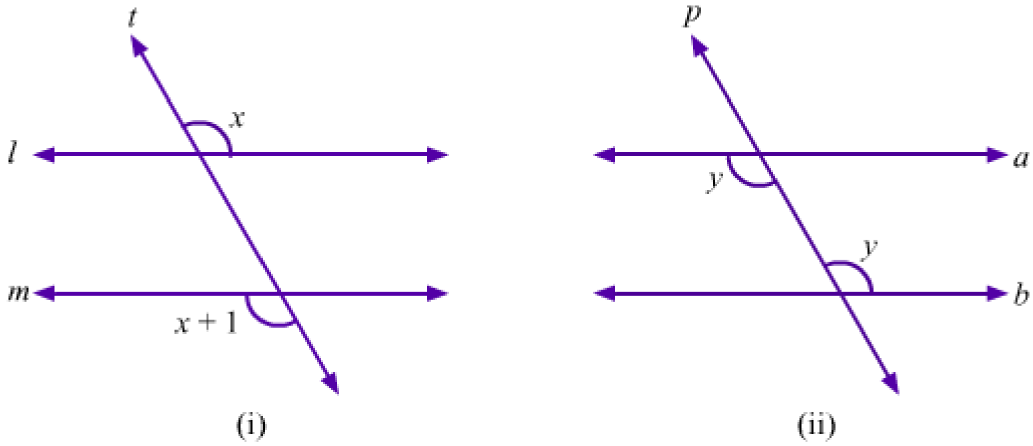


In the given figure, the sum of the interior angles on the right side of the transversal *n* is 170° which is less than two right angles or 180°. So, lines *l* and *m* will intersect when extended indefinitely on the same side as the given interior angles.

Solved Examples

Easy

Example 1: **In which case are the lines cut by the transversal parallel to each other?**

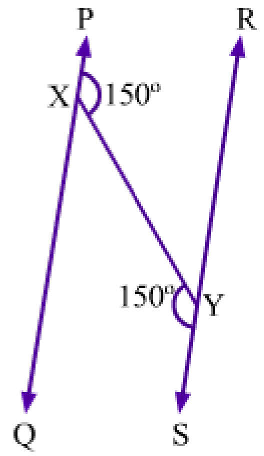


Solution:

In figure (i), the alternate exterior angles formed by the transversal t with lines l and m are not equal as $x \neq x + 1$. Therefore, lines l and m are not parallel to each other.

In figure (ii), the alternate interior angles formed by the transversal p with lines a and b are equal as $y = y$. Therefore, line a is parallel to line b.

Example 2: **Is PQ parallel to RS in the given figure?**



Solution:

In the figure, we are given ∠PXY and ∠SYX.

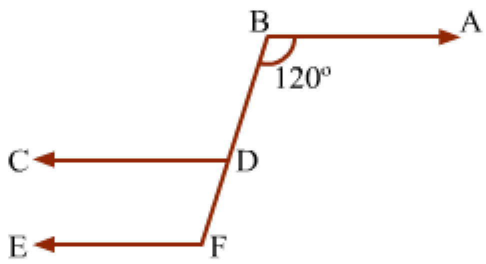
These are the alternate interior angles made by PQ and RS with the transversal XY.

Now, ∠PXY = ∠SYX = 150°

Therefore, by the converse of the alternate angles axiom, we have PQ||RS.

Medium

Example 1: **In the given figure, AB is parallel to CD and CD is parallel to EF. It is given that**
∠ABD = 120°. Show that AB is parallel to EF.



Solution:

It is given that ∠ABD = 120°.

Also, AB is parallel to CD.

∴ ∠BDC = ∠ABD = 120° (Pair of alternate interior angles between parallel lines)

We know that CD is parallel to EF.

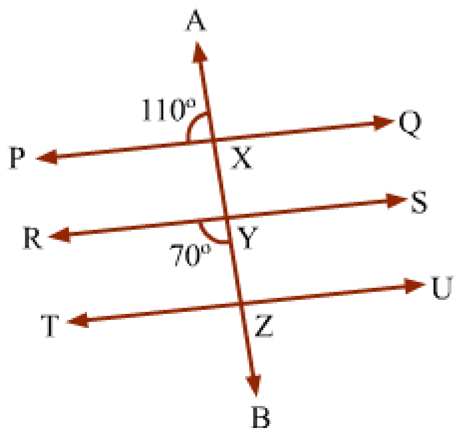
∴ ∠BFE = ∠BDC = 120° (By the corresponding angles axiom)

⇒ ∠BFE = ∠ABD

Now, ∠ABD and ∠BFE form a pair of alternate interior angles with respect to lines AB and EF.

Thus, by using the converse of the alternate angles axiom, we obtain AB||EF.

Example 2: **If RS||TU, then prove that PQ||TU.**



Solution:

We are given that RS||TU and AB is the transversal.

Now, the interior angles on the same side of the transversal are supplementary.

So, ∠RYB + ∠TZA = 180°

⇒ ∠TZA = 180° − 70°

⇒ ∴ ∠TZA = 110°

It is given that ∠PXA = 110°.

∴ ∠QXB = 110° (because ∠PXA and ∠QXB are vertically opposite angles)

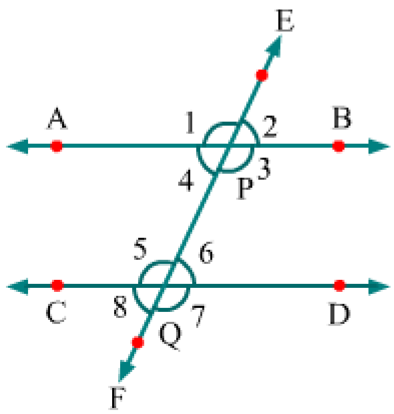
⇒ ∠QXB = ∠TZA

Now, ∠QXB and ∠TZA form a pair of alternate interior angles with respect to lines PQ and TU.

Thus, by using the converse of the alternate angles axiom, we obtain PQ||TU.

Hard

Example 1: **In the following figure, AB and CD are parallel to each other and EF is the transversal intersecting AB and CD at points P and Q respectively. If ∠APE = 110°, then find all the angles formed at points P and Q.**



Solution:

In the given figure, we have the angles at points P and Q numbered from 1 to 8. Also, we have ∠APE = ∠1 = 110°.

Now,

∠3 = ∠1 = 110° (Vertically opposite angles)

∠5 = ∠3 = 110° (Alternate interior angles between parallel lines)

∠7 = ∠5 = 110° (Vertically opposite angles)

∠1 and ∠2 form a linear pair.

So, ∠1 + ∠2 = 180°

⇒ 110° + ∠2 = 180°

⇒ ∴ ∠2 = 70°

Now,

∠4 = ∠2 = 70° (Vertically opposite angles)

∠6 = ∠4 = 70° (Alternate interior angles between parallel lines)

∠8 = ∠6 = 70° (Vertically opposite angles)

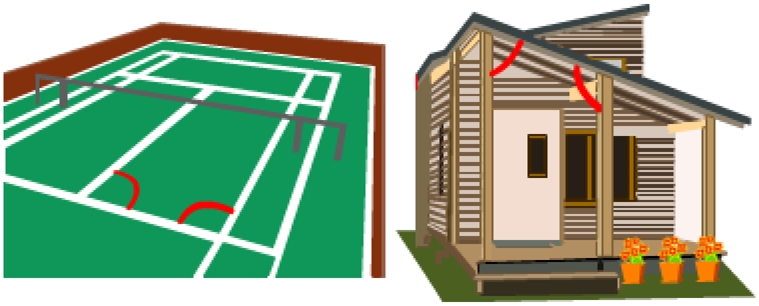
Thus, we have the angles around points P and Q as follows:

∠1 = ∠3 = ∠5 = ∠7 = 110° and ∠2 = ∠4 = ∠6 = ∠8 = 70°

Interior Angles on The Same Side of The Transversal

Experiencing Interior Angles on the Same Side of a Transversal

Consider the following figures of a tennis court and a house.

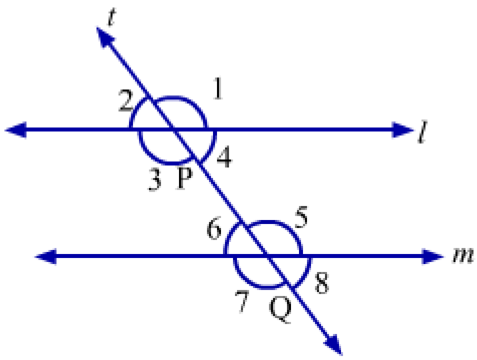


Transversals on parallel lines can be seen easily in the two figures. In each figure, the marked angles are interior angles lying on the same side of a transversal. Examples of such angles can be seen in many of the things that surround us.

In this lesson, we will discuss the property of interior angles on the same side of a transversal and solve some problems based on the same.

Property of Interior Angles on the Same Side of a Transversal

Consider the given figure.



In the figure, the transversal *t* intersects two parallel lines *l* and *m* at points P and Q respectively. Four angles are formed around each point. These angles have been numbered from 1 to 8.

Now, ∠3 and ∠6 form a pair of interior angles lying on the same side of the transversal *t*. ∠4 and ∠5 is another such pair of interior angles. The property exhibited by these types of angles is stated as follows:

If a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

∴ $\angle 3 + \angle 6 = 180°$ and $\angle 4 + \angle 5 = 180°$

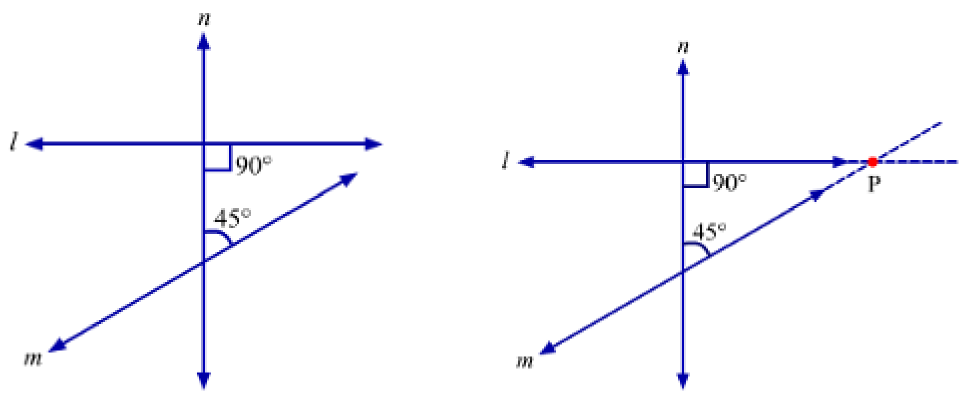
The converse of this property is also true. It states that:

If a transversal intersects two lines such that the interior angles on the same side of the transversal are supplementary, then the lines intersected by the transversal are parallel.

Whiz Kid

If the sum of the interior angles on the same side of a transversal is less than two right angles or 180°, then the lines cut by the transversal must intersect when extended along that side of the transversal.

For example:

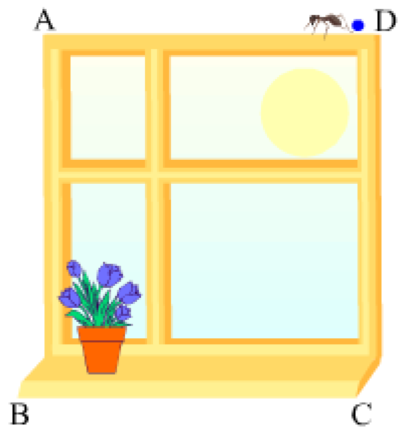


In first figure, the sum of the interior angles on the right side of transversal *n* is 90° + 45° = 135° < 180°. So, lines *l* and *m* will intersect when extended along the right side of the transversal *n*, as is shown in second figure.

Solved Examples

Easy

Example 1: An ant is moving on a window frame by following the path DABC. If edge AD is parallel to edge BC, then what angle will the ant have to move along in order to reach point C from point D?



Solution:

In the window frame, edges AD and BC are parallel to each other and edge AB is the transversal. So,∠A and ∠B are interior angles lying on the same side of the transversal AB.

From point D to point C, the ant will move along a total angle that is the sum of ∠A and ∠B.

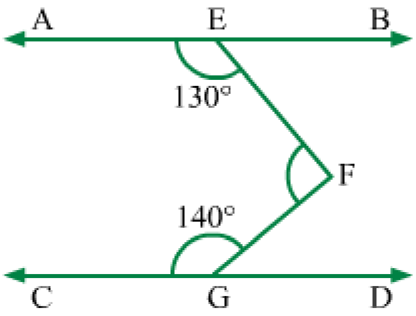
We know that if a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

∴ $\angle A + \angle B = 180°$

Hence, the ant will have to move along an angle of 180° to reach point C from point D.

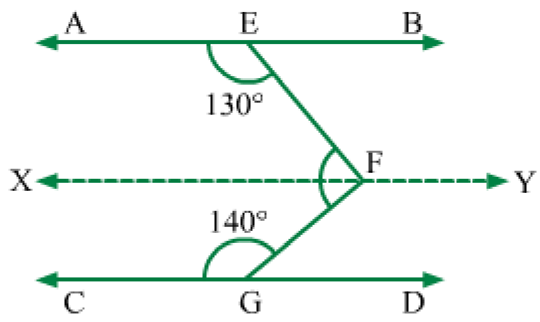
Medium

Example 1: In the given figure, AB and CD are parallel lines. Find the measure of ∠EFG.



Solution:

Construction: Draw a line XY passing through point F and parallel to lines AB and CD.



Now, we have EF as the transversal on the parallel lines AB and XY. Similarly, we have GF as the transversal on the parallel lines CD and XY.

∠AEF and ∠EFX are interior angles on the same side of the transversal EF.

∴ ∠AEF + ∠EFX = 180°

⇒ 130° + ∠EFX = 180°

⇒ ∠EFX = 180° − 130°

⇒ ∴ ∠EFX = 50°

Similarly,

∠CGF + ∠GFX = 180°

⇒ 140° + ∠GFX = 180°

⇒ ∠GFX = 180° − 140°

⇒ ∴ ∠GFX = 40°

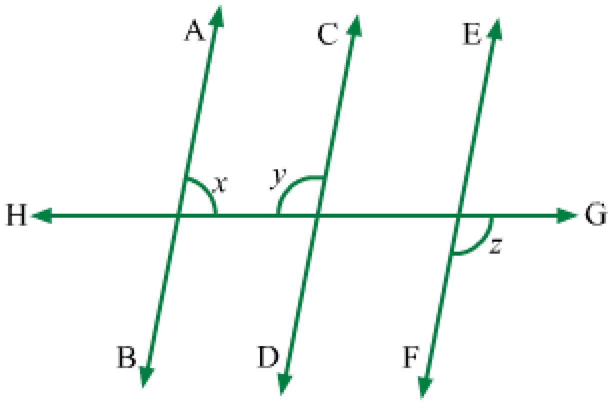
From the figure, we have:

∠EFG = ∠EFX + ∠GFX

⇒ ∠EFG = 40° + 50°

⇒ ∴ ∠EFG = 90°

Example 2: **In the given figure, AB||CD, CD||EF and** $x : z = 2 : 3$**. What are the measures of** x**,** y **and** z**?**



Solution:

It is given that AB||CD, CD||EF and GH is the transversal on these pairs of parallel lines.

Also, $x : z = 2 : 3$

$$\Rightarrow \frac{x}{z} = \frac{2}{3}$$

Let $x = 2a$ and $z = 3a$

Now,

$y = z$ (Exterior alternate angles formed by GH on CD and EF)

$x + y = 180°$ (Interior angles on the same side of GH)

$\Rightarrow x + z = 180°$

$\Rightarrow 2a + 3a = 180°$

$\Rightarrow 5a = 180°$

$\Rightarrow \therefore a = 36°$

So, we have $x = 2 \times 36° = 72°$ and $z = 3 \times 36° = 108°$

Also, $y = z = 108°$

Activity Time

Follow these steps to verify the property of interior angles lying on the same side of a transversal.

• Take a chart paper.

• Draw two parallel lines.

• Draw four or five transversals, each intersecting the parallel lines at two different points.

• Make a list of the pairs of interior angles formed on the same side of each transversal.

• Measure each angle in the list using a protractor.

• Find the sum of the angles in each listed pair of angles.

• Check whether each pair of angles consists of supplementary angles or not.

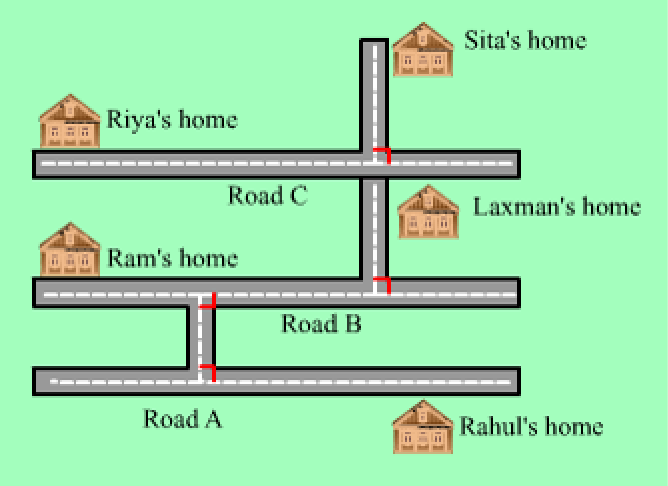
• Write the result common to all the transversals.

This activity proves the property that interior angles on the same side of a transversal lying on two parallel lines are supplementary.

Theorems on Coplanar Lines

Lines Parallel to the Same Line: In Real Life

Observe the following map.



In the map, roads A and B are parallel to each other. Similarly, roads B and C are parallel to each other. It can be seen that if Rahul moves along Road A, he reaches neither Ram's home nor Riya's home. Similarly, if Riya moves along Road C, she reaches neither Ram's home nor Rahul's home. Ram, too, cannot reach Riya's home or Rahul's home simply by moving along Road B. In order to reach one another's homes, they need to move through the transversal roads. This is because like parallel lines parallel roads never intersect one another.

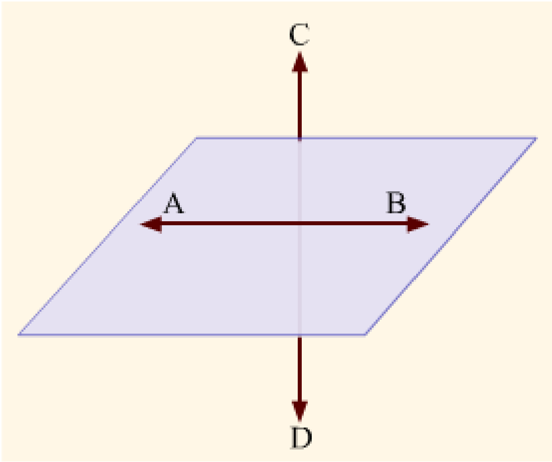
Since Riya, Rahul and Ram are unable to reach one another's homes by using roads A, B and C individually, we can conclude that these roads don't intersect anywhere. In other words, the three roads are parallel to one another.

Initially, we considered only two pairs of parallel roads, but finally concluded that all the three roads are parallel to one another. This shows that there is a relation between two lines parallel to the same line. We will discuss this property of lines parallel to the same line and solve some examples related to it.

Whiz Kid

Lines that are neither parallel nor intersecting are known as **skew lines**. Such lines lie in different planes.

For example:

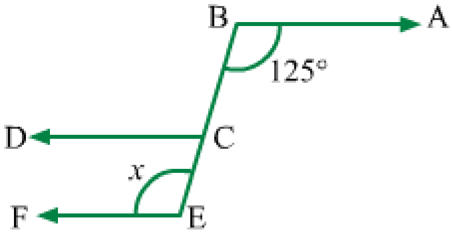


In the given figure, lines AB and CD are neither parallel nor intersecting because they lie in different planes. So, AB and CD are skew lines.

Solved Examples

Easy

Example 1: In the given figure, AB is parallel to CD and CD is parallel to EF. Find the value of *x*.



Solution:

It is given that AB||CD and CD||EF.

We know that lines parallel to the same line are parallel to one another.

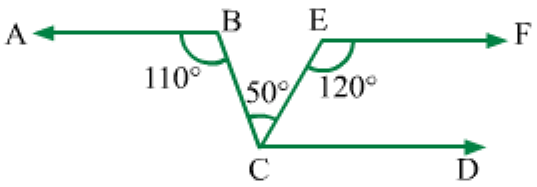
∴ AB||EF

Now, BE is the transversal lying on the parallel lines AB and EF.

So, x = ∠ABE = 125° (Alternate Interior angles)

Medium

Example 1: In the given figure, CD is parallel to EF. Show that AB and EF are also parallel.



Solution:

It is given that CD||EF and EC is the transversal on these parallel lines.

So, ∠ECD + ∠CEF = 180° (Interior angles on the same side of the transversal EC)

⇒ ∠ECD + 120° = 180°

⇒ ∴ ∠ECD = 180° − 120° = 60°

Now, ∠BCD = ∠BCE + ∠ECD

⇒ ∠BCD = 50° + 60° = 110°

∠BCD and ∠ABC form a pair of interior opposite angles with respect to AB and CD intersected by the transversal BC.

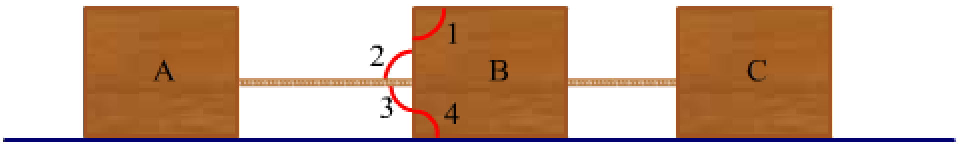
Also, ∠BCD = ∠ABC = 110°

∴ AB||CD (By the converse of the alternate angles axiom)

Since AB and EF are lines parallel to the same line CD, we have:

AB||EF

Example 2: In the given figure, there are three blocks A, B and C tied by a rope. If ∠1 = ∠2 and ∠3 = ∠4, then what can be said about the parallelism of the horizontal sides of block B?



Solution:

It is given that ∠1 = ∠2 and ∠3 = ∠4.

In the given figure, ∠1 and ∠2 are alternate angles. They are also equal.

Therefore, by the converse of the alternate angles axiom, the rope is parallel to the top side of block B.

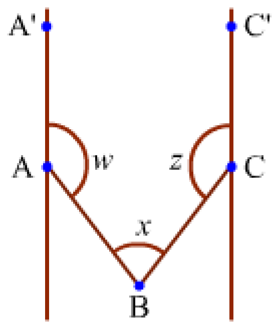
Similarly, ∠3 and ∠4 are equal alternate angles.

Therefore, by the converse of the alternate angles axiom, the rope is also parallel to the bottom side of block B.

We know that *lines parallel to the same line are parallel to one another*. Since the horizontal sides of block B are parallel to the rope, they are also parallel to each other.

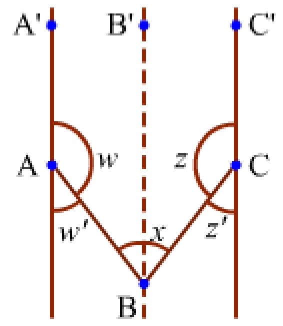
Hard

Example 1: In the figure, AA' is parallel to CC'. If w = 135° and z = 147°, then find x.



Solution:

Construction: Draw a line BB' parallel to AA' and CC'.



Thus, we obtain AB as the transversal on AA' and BB'; and BC as the transversal on BB' and CC'.

Now,

x = ∠ABC = ∠ABB' + ∠CBB'

w' and ∠ABB' are alternate interior angles between parallel lines.

∴ ∠ABB' = w'

Similarly, z' and ∠CBB' are alternate interior angles between parallel lines.

∴ ∠CBB' = z'

w and w' form a linear pair; thus, they are supplementary.

$w + w' = 180°$

$\Rightarrow w' = 180° - w$

$\Rightarrow w' = 180° - 135°$

$\Rightarrow \therefore w' = 45°$

⇒ ∠ABB' = 45°

Similarly, z and z' form a linear pair; thus, they are supplementary.

$z + z' = 180°$

$\Rightarrow z' = 180° - z$

⇒ $z' = 180° - 147°$

⇒ ∴ $z' = 33°$

⇒ ∠CBB' = 33°

So, x = ∠ABC = ∠ABB' + ∠CBB' = 45° + 33° = 78°

Corollaries Related to Property

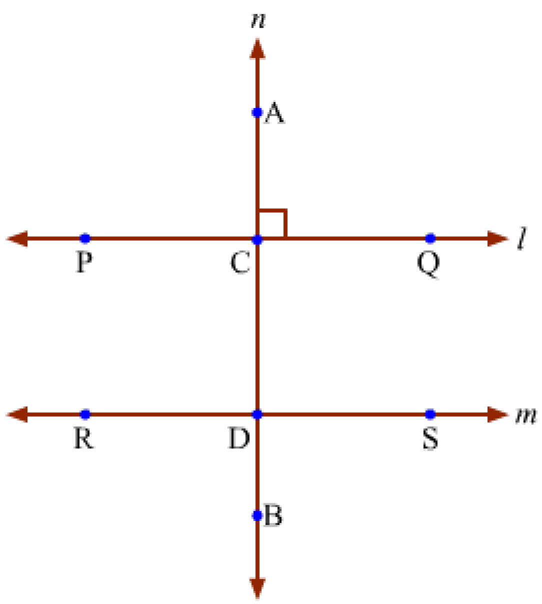
If a line is coplanar with two given parallel lines and it is perpendicular to one of them then it is perpendicular to the other also.

Let us prove the above statement.

Given: Three coplanar lines l, m and n such that $l \parallel m$ and $n \perp l$

To prove: $n \perp m$

Proof:



∠ACQ = ∠CDS ...(1) (Corresponding angles)

∠ACQ = 90° ...(2) ($n \perp l$)

From (1) and (2), we obtain

∠CDS = 90°

∴ $n \perp m$

Corollaries Related to Property

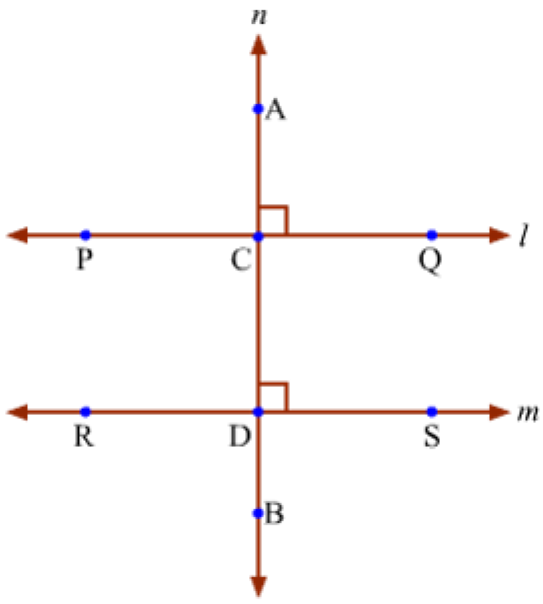
If a line is coplanar with two given lines and it is perpendicular to both of them then the given lines are parallel to each other.

Let us prove the above statement.

Given: Three coplanar lines l, m and n such that $n \perp l$ and $n \perp m$

To prove: $l \parallel m$

Proof:



$n \perp l$ (Given)

$\therefore \angle ACQ = 90°$ ...(1)

$n \perp m$ (Given)

$\therefore \angle CDS = 90°$ ...(2)

From (1) and (2), we obtain

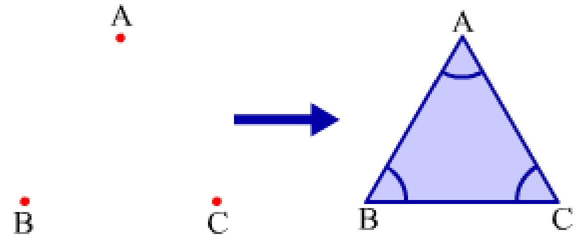
$\angle ACQ = \angle CDS$

$\therefore l \parallel m$ (Using corresponding angles axiom)

Angle Sum Property of Triangles

Angle Sum Property of Triangles

If we join any three non-collinear points in a plane, then we **get a triangle. There are three angles in a triangle.**

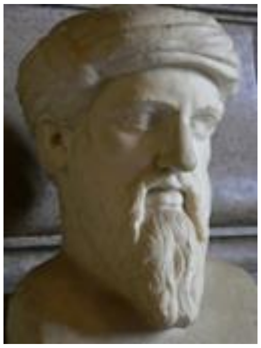


The sum of the three **interior angles** of a triangle is 180° and this property of a triangle is known as the angle sum property. This property holds true for all types of triangles, i.e., **acute-angled triangles**, **obtuse-angled triangles** and **right-angled triangles**. The angle sum property was identified by the Pythagorean school of Greek mathematicians (or the Pythagoreans) and proved by Euclid.

We will study the proof of the angle sum property of triangles and then solve some examples based on this property.

Know Your Scientist

Pythagoras



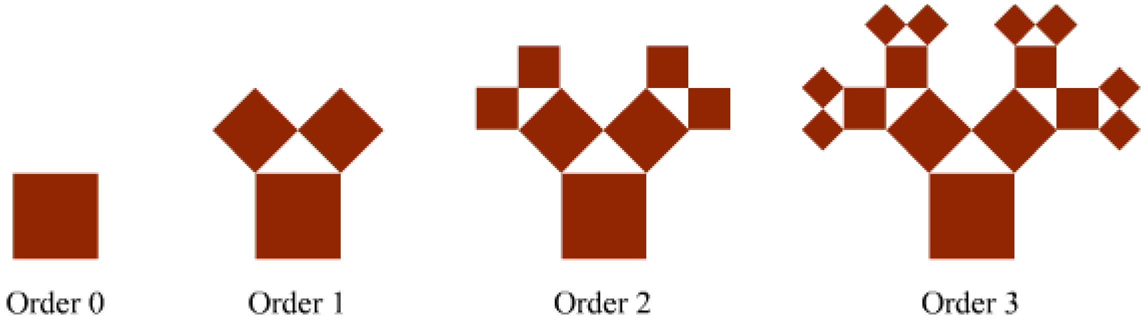
Pythagoras (570 BC–495 BC) was a great Greek mathematician and philosopher, often described as the first pure mathematician. He was born on the island of Samos and is best known for the Pythagoras theorem about right-angled triangles. He also made influential contributions to philosophy and religious teaching. He led a society that was part religious and part scientific. This society followed a code of secrecy, which is the reason why a sense of mystery surrounds the figure of Pythagoras.

Euclid

Euclid of Alexandria(325 BC–265 BC) was a great Greek mathematician. He is referred to as 'the father of geometry'. Euclid taught at Alexandria during the reign of Ptolemy I, who ruled Egypt from 323 BC to 285 BC. Euclid wrote a series of books which are collectively known as the *Elements*.It is considered one of the most influential works in the history of mathematics. The *Elements* served as the main textbook for teaching mathematics (especially geometry) from the time of its publication up until the early 20th century. In the *Elements*, Euclid defined most of the basic geometrical figures and deduced the principles of geometry through different sets of axioms.

Did You Know?

In 1942, a Dutch mathematics teacher Albert E. Bosman invented a plane fractal constructed from a square. He named it the Pythagoras tree because of the presence of right-angled triangles in the figure.



Construction process of Pythagoras tree

Facts about the Angle Sum Property

An important fact deduced through the angle sum property of triangles is that ***there can be no triangle with two right angles or two obtuse angles*****. This fact can be proved as is shown.**

Consider a ΔABC such that ∠A = 90° and ∠B = 90°.

According to the angle sum property, we have:

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + 90° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 180°$

$\Rightarrow \angle C = 0°$

However, the above is not possible. So, ΔABC (or any other triangle) cannot have two right angles.

Similarly, we can prove that a triangle cannot have two obtuse angles.

Whiz Kid

Relationship between the side lengths and the angle measurements of a triangle

• The largest interior angle is **opposite** the largest side.

• The smallest interior angle is **opposite** the smallest side.

• The middle-sized interior angle is **opposite** the middle-sized side.

Facts about the Angle Sum Property

By the angle sum property, we can deduce the fact that ***there can be no triangle with all angles less than or greater than* 60°.** This fact can be proved as is shown.

Consider a **ΔABC** with all angles equal to 59°.

According to the angle sum property, we should have **$\angle A + \angle B + \angle C = 180°$.**

By adding the given angles, we obtain:

$59° + 59° + 59°$ **$= 177° \neq 180°$**

Since ΔABC does not satisfy the angle sum property, it cannot exist.

Now, consider a **ΔABC** with all angles equal to 61°.

According to the angle sum property, we should have **$\angle A + \angle B + \angle C = 180°$.**

By adding the given angles, we obtain:

61° **+** 61° **+** 61° **= 183° ≠ 180°**

Since ΔABC does not satisfy the angle sum property, it cannot exist.

Thus, we have proved that a triangle cannot have all angles less than or greater than 60°.

Whiz Kid

Sum of the interior angles of an n-sided polygon = $(n - 2) \times 180°$

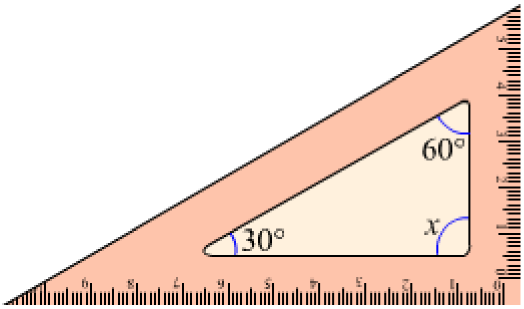
For example:

Sum of the interior angles of a 6-sided polygon = $(6 - 2) \times 180° = 720°$

Solved Examples

Easy

Example 1: Find the measurement of *x* in the following figure.



Solution:

We know that the sum of the three angles of a triangle is 180°.

So, we have:

$30° + 60° + x = 180°$

$\Rightarrow 90° + x = 180°$

$\Rightarrow x = 90°$

Example 2: If the angles of a triangle are in the ratio 1 : 3 : 5, then what is the measure of each angle?

Solution:

It is given that the angles of the triangle are in the ratio 1 : 3 : 5.

Let the angles be x, $3x$ and $5x$.

Now, $x + 3x + 5x = 180°$ (By the angle sum property of triangles)

$\Rightarrow 9x = 180°$

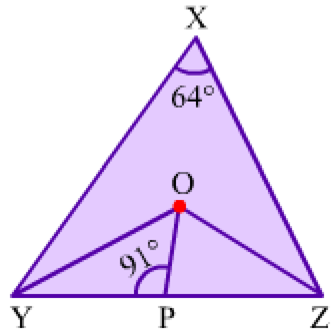
$\Rightarrow x = 20°$

So, $3x = 3 \times 20° = 60°$ and $5x = 5 \times 20° = 100°$

Thus, the measures of the angles of the triangle are 20°, 60° and 100°.

Medium

Example 1: In the given ΔXYZ, YO, ZO and PO are the respective bisectors of ∠XYZ, ∠XZY and ∠YOZ. Find the measure of ∠OYX.



Solution:

As per the relation between the vertex angle and the angles made by the bisectors of the remaining two angles, we have:

$\angle YOZ = 90^\circ + \frac{1}{2}\angle YXZ$

$\Rightarrow \angle YOZ = 90^\circ + \frac{1}{2} \times 64^\circ$

$\Rightarrow \angle YOZ = 90^\circ + 32^\circ$

$\Rightarrow \angle YOZ = 122^\circ$

In ΔOYP, we have:

$\angle YOP = \frac{1}{2} \angle YOZ$ (&because PO is the bisector of ∠YOZ)

$\Rightarrow \angle YOP = \frac{1}{2} \times 122^\circ$

$\Rightarrow \angle YOP = 61^\circ$

Again in ΔOYP, we have:

$\angle YOP + \angle OPY + \angle PYO = 180^\circ$ (By the angle sum property)

$\Rightarrow 61^\circ + 91^\circ + \angle PYO = 180^\circ$

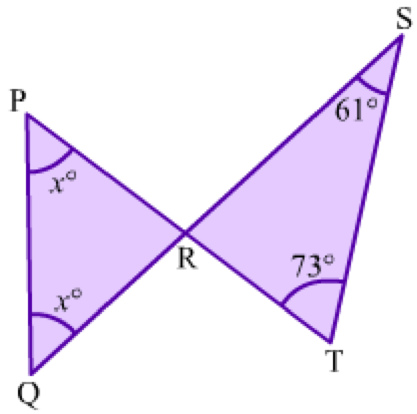
$\Rightarrow \angle PYO = 180^\circ - 152^\circ$

$\Rightarrow \angle PYO = 28^\circ$

We know that YO is the bisector of ∠XYZ.

So, $\angle OYX = \angle PYO = 28^\circ$

Example 2: Find the value of *x* in the given figure.



Solution:

Using the angle sum property in ΔRST, we obtain:

$\angle RST + \angle RTS + \angle SRT = 180°$

$\Rightarrow 61° + 73° + \angle SRT = 180°$

$\Rightarrow 134° + \angle SRT = 180°$

$\Rightarrow \angle SRT = 180° - 134°$

$\Rightarrow \angle SRT = 46°$

In ΔRST and ΔRPQ, we have:

$\angle PRQ = \angle SRT = 46°$ (Vertically opposite angles)

Using the angle sum property in ΔRPQ, we obtain:

$\angle RPQ + \angle RQP + \angle PRQ = 180°$

$\Rightarrow x° + x° + 46° = 180°$

$\Rightarrow 2x° + 46° = 180°$

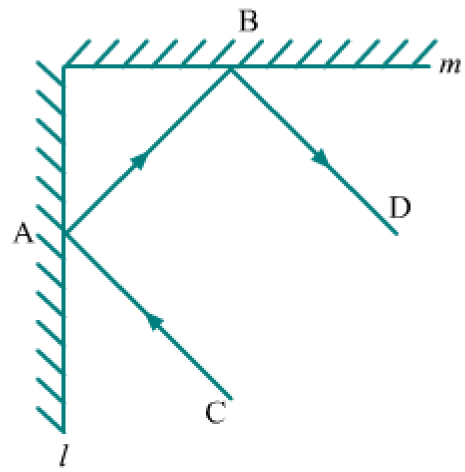
$\Rightarrow 2x° = 180° - 46°$

$\Rightarrow 2x° = 134°$

$\Rightarrow x = 67$

Hard

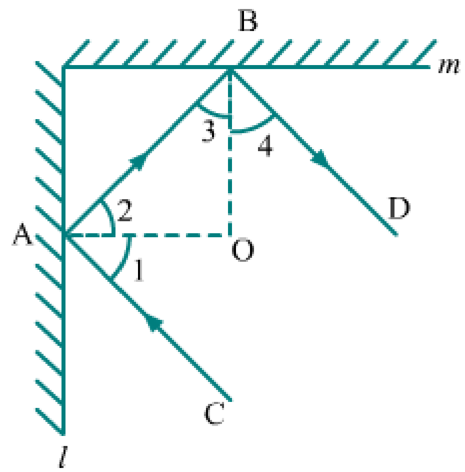
Example 1: **In the figure, *l* and *m* are two plane mirrors placed perpendicular to each other. Show that the incident ray CA is parallel to the reflected ray BD.**



Solution:

It is given that mirrors *l* and *m* are perpendicular to each other.

Construction: Draw two perpendiculars OA and OB to *l* and *m* respectively. Mark the angles made by these perpendiculars with the incident and reflected rays as is shown.



OA ⊥ OB

∴ **∠BOA = 90°**

In ΔBOA, we have:

$\angle 2 + \angle 3 + \angle BOA = 180°$

$\Rightarrow \angle 2 + \angle 3 + 90° = 180°$

$\Rightarrow \angle 2 + \angle 3 = 90°$

$\Rightarrow 2(\angle 2 + \angle 3) = 180°$

$\Rightarrow 2\angle 2 + 2\angle 3 = 180°$ **... (1)**

We know that:

Angle of incidence = Angle of reflection

For mirror *l*, ∠1 is the angle of incidence and ∠2 is the angle of reflection.

$\therefore \angle 1 = \angle 2$

For mirror *m*, ∠3 is the angle of incidence and ∠4 is the angle of reflection.

$\therefore \angle 3 = \angle 4$

So, by using equation 1, we get:

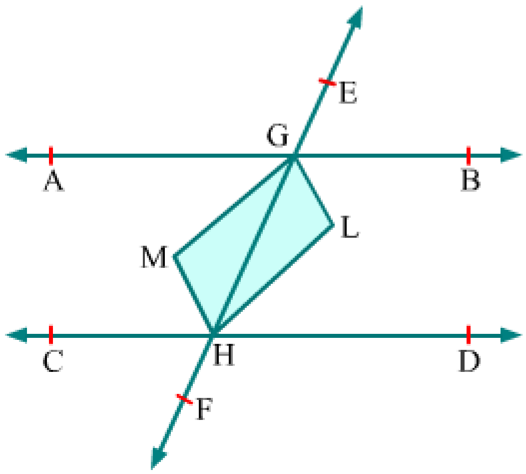
$(\angle 2 + \angle 2) + (\angle 3 + \angle 3) = 180°$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°$ **(**Since ∠1 = ∠2 and ∠3 = ∠4**)**

$\Rightarrow \angle CAB + \angle ABD = 180°$

Since ∠CAB and ∠ABD are interior angles on the same side of transversal AB and their sum is 180°**, lines CA and BD must be parallel to each other. Therefore, the incident ray CA is parallel to the reflected ray BD.**

Example 2: **In the given figure, AB is parallel to CD; GM, HM, GL and HL are the bisectors of the two pairs of interior angles. Prove that ∠GLH = 90°.**



Solution:

From the figure, we have:

∠AGH = ∠DHG (Alternate interior angles)

$\Rightarrow \frac{1}{2}\angle AGH = \frac{1}{2}\angle DHG$

⇒ ∠HGM = ∠GHL (&because GM bisects ∠AGH and HL bisects ∠DHG)

It can be said that lines GM and HL are intersected by the transversal GH at G and H respectively such that the alternate interior angles are equal, i.e., ∠HGM = ∠GHL.

∴ GM||HL

Similarly, we can prove that GL||HM. So, GMHL is a parallelogram.

We know that AB||CD and EF is the transversal.

∴ ∠BGH + ∠DHG = 180° (Interior angles on the same side of a transversal)

$\Rightarrow \frac{1}{2}\angle BGH + \frac{1}{2}\angle DHG = 90°$

Since GL bisects ∠BGH and HL bisects ∠DHG, we obtain:

∠LGH + ∠GHL = 90° ... (1)

Also, ∠LGH + ∠GHL + ∠GLH = 180° (By the angle sum property)

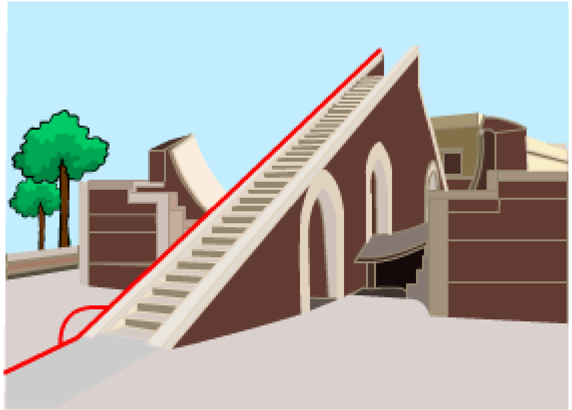
∴ 90° + ∠GLH = 180° (Using equation 1)

⇒ ∠GLH = 90°

Exterior Angle Property of Triangles

Exterior Angles in Real Life

Look at the triangular structure in the figure.

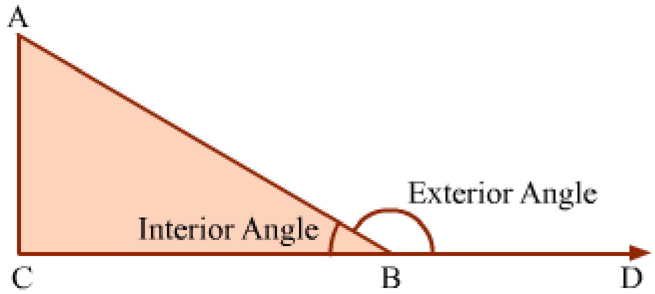


In the given figure, an open angle formed by the edge of the triangular structure with the horizontal plane is marked. This angle lies outside the triangle. Such angles are known as **exterior angles**.

In this lesson, we will study about exterior angles of triangles and the theorem based on them.

Exterior Angles of Triangles

Look at the triangle shown.



It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

An exterior angle of a triangle can be defined as follows:

The angle formed by a side of a triangle with an extended adjacent side is called an exterior angle of the triangle.

It can be seen that exterior ∠ABD forms linear pair with interior ∠ABC of **Δ**ABC. The other two interior angles of the triangle such as ∠ACB and ∠CAB do not form linear pair with ∠ABD.

Such angles are known as the **remote interior angles** of an exterior angle.

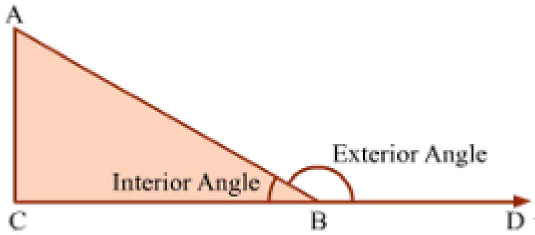
So, ∠ACB and ∠CAB are remote interior angles of exterior ∠ABD.

Corollary Related to Exterior Angle Theorem

There is a corollary related to exterior angle theorem which states that:

An exterior angle is always greater than each of its remote interior angles.

Let us prove this corollary with the help of **Δ**ABC shown in the figure.



Here, ∠ABD is an exterior angle of the triangle and its interior opposite or remote interior angles are ∠ACB and ∠CAB.

In a triangle, no interior angle can be zero angle or straight angle.

Thus, 0° <∠ABC < 180°, 0° < ∠ACB < 180° and 0° < ∠CAB < 180°

Now, ∠ABC < 180°

∴ 180° – ∠ABC > 0°

⇒ ∠ABD > 0°

In triangle **Δ**ABC, we have

∠ABD = ∠ACB +∠CAB (By exterior angle theorem)

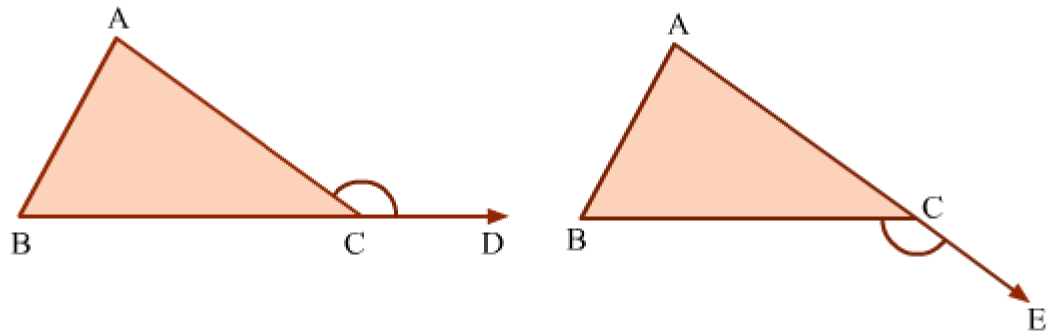
And, ∠ABD > 0, ∠ACB > 0° and ∠CAB > 0° (Property of triangle)

Therefore, ∠ABD > ∠ACB and∠ABD >∠CAB

Thus, an exterior angle is always greater than each of its remote interior angles.

Two Exterior Angles at the Same Vertex are Equal

At any vertex, two exterior angles can be drawn by extending each of the two sides forming that vertex. These exterior angles are always of equal measure. Let us prove this using the ΔABC shown in the figure.



The figure clearly shows that two exterior angles can be drawn at vertex C—one by producing BC up to point D and the other by producing AC up to point E. The exterior angles thus obtained are ∠ACD and ∠BCE.

According to the exterior angle theorem, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles of the triangle.

∴ ∠ACD = ∠ABC + ∠BAC ... (1)

And, ∠BCE = ∠ABC + ∠BAC ... (2)

Using equations 1 and 2, we get:

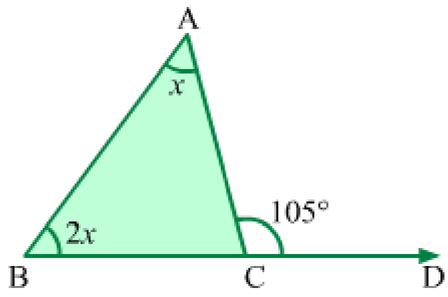
∠ACD = ∠BCE

So, we can conclude that two exterior angles can be drawn at any vertex. The two angles thus drawn have an equal measure and are equal to the sum of the two opposite interior angles.

Solved Examples

Easy

Example 1: Find the value of *x* in the given figure.



Solution:

According to the exterior angle property of triangles, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles of the triangle.

So, we have:

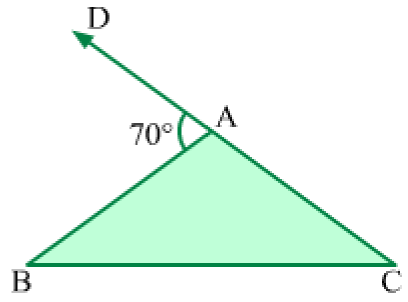
$x + 2x = 105°$

$\Rightarrow 3x = 105°$

On dividing both sides of the equation by 3, we obtain:

$$\frac{3x}{3} = \frac{105°}{3}$$
$$\Rightarrow x = 35°$$

Example 2: If ∠ABC = ∠ACB in ΔABC, then find the measure of ∠ABC.



Solution:

∠ABC and ∠ACB are interior angles opposite to the exterior angle at vertex A, i.e., ∠BAD.

Therefore, by the exterior angle property of triangles, we obtain:

$\angle ABC + \angle ACB = \angle BAD$

$\Rightarrow \angle ABC + \angle ACB = 70°$

It is given that $\angle ABC = \angle ACB$

So, we obtain:

$\angle ABC + \angle ABC = 70°$

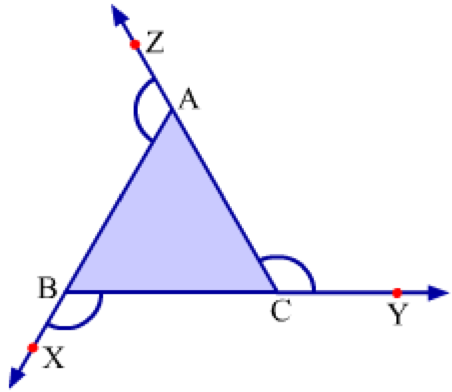
$\Rightarrow 2\angle ABC = 70°$

On dividing both sides of the equation by 2, we obtain:

$$\frac{2\angle ABC}{2} = \frac{70°}{2}$$
$$\Rightarrow \angle ABC = 35°$$

Medium

Example 1: The sides AB, BC and CA of ΔABC are produced up to points X, Y and Z respectively. Find the sum of the three exterior angles so formed.



Solution:

Using the exterior angle property, we obtain:

$\angle BAZ = \angle ABC + \angle ACB$ ... (1)

$\angle CBX = \angle BAC + \angle ACB$ ... (2)

$\angle ACY = \angle BAC + \angle ABC$ ... (3)

On adding equations 1, 2 and 3, we obtain:

∠BAZ + ∠CBX + ∠ACY = ∠ABC + ∠ACB + ∠BAC + ∠ACB + ∠BAC + ∠ABC

⇒ ∠BAZ + ∠CBX + ∠ACY = 2(∠ABC + ∠ACB + ∠BAC)

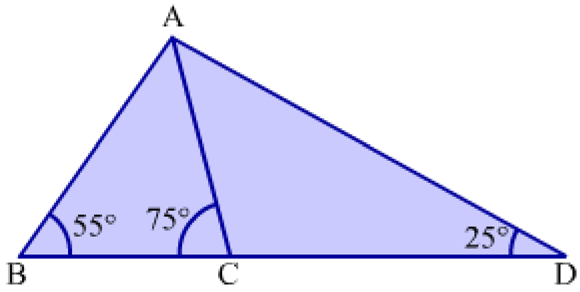
According to the angle sum property of triangles, we have:

∠ABC + ∠ACB + ∠BAC = 180°

∴ ∠BAZ + ∠CBX + ∠ACY = 2 × 180° = 360°

Thus, the sum of the three exterior angles is 360°.

Example 2: Show that AC is the bisector of ∠BAD in the given figure.



Solution:

On applying the angle sum property in ΔABC, we get:

∠BAC + ∠ABC + ∠ACB = 180°

⇒ ∠BAC + 55° + 75° = 180°

⇒ ∠BAC = 180° − 130°

⇒ ∠BAC = 50° ... (1)

Now, by using the exterior angle property, we get:

∠ACB = ∠ADC + ∠CAD

⇒ 75° = 25° + ∠CAD

⇒ ∠CAD = 75° − 25°

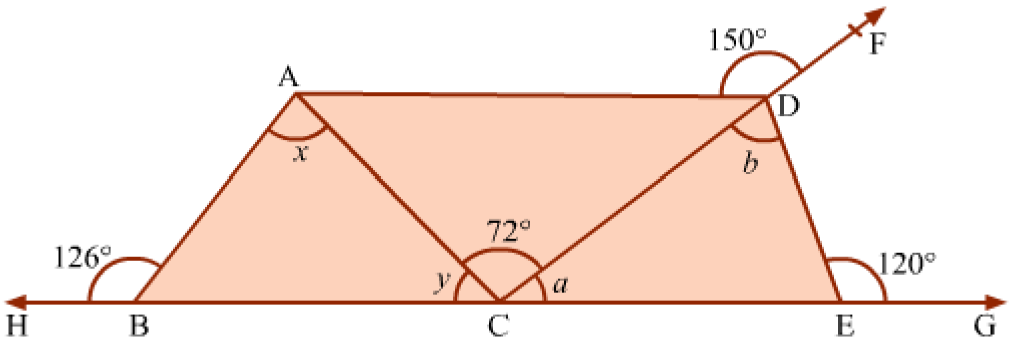
⇒ ∠CAD = 50°

We know that ∠BAC + ∠CAD = ∠BAD. We have found ∠BAC = ∠CAD = 50°.

Thus, AC is the bisector of ∠BAD.

Hard

Example 1: **If AD||BE in the given figure, then find the values of *a*, *b*, *x* and *y*.**



Solution:

From the figure, we have:

∠ADC + ∠ADF = 180° (Linear pair of angles)

⇒ ∠ADC + 150° = 180°

⇒ ∠ADC = 30°

Consider the parallel lines AD and BE and the transversal CF.

∠ADF = ∠DCB (Corresponding angles)

⇒ 150° = 72° + y

⇒ y = 78° ... (1)

Now, y + 72° + a = 180° (As they form line BCE)

⇒ 78° + 72° + a = 180° (Using equation 1)

⇒ a = 180° − 150°

⇒ $a = 30°$ ... (2)

Consider ΔCDE.

∠DEG = $a + b$ (Exterior angle property)

⇒ $120° = 30° + b$ (Using equation 2)

⇒ $b = 90°$

Now, consider ΔABC.

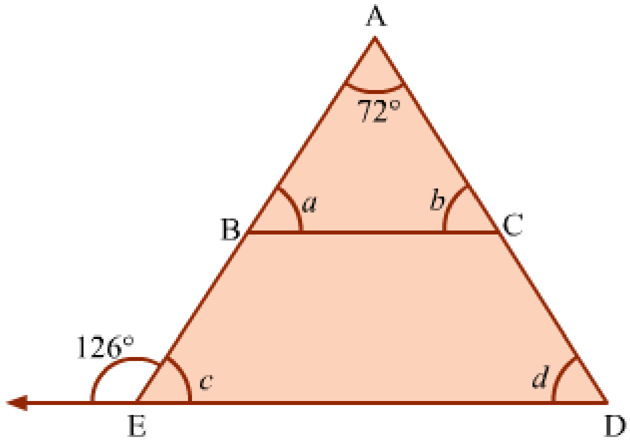
∠ABH = $x + y$ (Exterior angle property)

⇒ $126° = x + 78°$

⇒ $x = 48°$

Hence, $a = 30°$, $b = 90°$, $x = 48°$ and $y = 78°$.

Example 2: **ΔABC is placed atop trapezium EBCD in the given figure. Find the values of *a*, *b*, *c* and *d*.**



Solution:

The exterior angle at E forms a linear pair with c.

∴ $126° + c = 180°$

⇒ $c = 180° - 126°$

$\Rightarrow c = 54°$

On using the exterior angle property in ΔAED,we get:

$126° = 72° + d$

$\Rightarrow d = 126° - 72°$

$\Rightarrow d = 54°$

Since EBCD is a trapezium, BC is parallel to ED. Also, BE and CD are transversals lying on the two parallel lines.

So, we have:

$a = c = 54°$ (Pair of corresponding angles)

$b = d = 54°$ (Pair of corresponding angles)

Thus, $a = b = c = d = 54°$.